
Subject: D16 Contest

Posted by [Magic](#) on Mon, 10 Sep 2012 05:56:58 GMT

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Hello all,

As you know, the Kickstater launched by Impact! Miniatures to mass produced some of my Truncated Sphere dice was very successful. But during this Kickstater Tom from Impact! challenged me to create a D16 with numbers on faces (my own version had numbers on edges). I found what I think is the best solution for this problem.

But Tom and I are not fully satisfied with this design.

So now, it's my turn to challenge you! If you can find a better design than the one described bellow, Tom will offer you a Shapeways prepaid gift coupon of \$100.

The die must satisfy the following requirements:

- Must be a 16-sided die (this one was obvious)
- Must be a Truncated Sphere: this means that you must first design a polyhedron with all the faces at the same distance from the center, and then intersect it with a sphere of the appropriate size - not too small so that most of the disc forming the faces of the die will touch and not too big so that those faces remain circular and do not overlap.
- Must have numbers on faces: to achieve that, each face must have an opposite face that is parallel.
- Must be better than my solution: the faces must not be too small - 48° or more of separation (I will explain what it is) are recommended and the rounded part must be evenly distributed. This excludes the dipyramid for instance.

You can now participate to the contest, but I'd like to give you some extra information.

Here is a full description of my own solution, so that you do not submit the same design.

I describe my dice by the numbers of faces by layers and, for each layer, the angle Theta that is between the normal of the face with the vertical axis (or if you prefer: the angle between the face and the horizontal plane) and then, for each face of the layer, the angle Phi giving its orientation around the vertical axis (a kind of latitude).

My D16 has its 16 faces distributed on 5 layers: 1+4+6+4+1

- First and last layer: one horizontal face. Basically, the die has two poles (faces A1 and A2). Theta=0 and 180° , Phi is meaningless in this case.
- Second and 4th layer: 4 faces of type B. For the 1st layer I have Theta is 49.627° and Phi= $\pm 33.4258^\circ$ and $\pm 146.5742^\circ$ (the sum of both values is 180°). The 4th layer is

symmetric relatively to the horizontal plane (the equator).

- 3rd layer, the equator. Obviously $\Theta=90^\circ$, and there are 2 kinds of faces. I have 4 faces of kind C for which $\Phi=\pm 65.1865^\circ$ and $\pm 114.8135^\circ$ (the sum of both values is 180°) and 2 faces of kind E ($\Phi=0^\circ$ and 180°).

Of course if you decide to put the faces of kind D at the pole, the new repartition of layer is $1+4+2+2+2+4+1$, and if you put two C faces at the pole it is $2+4+4+4+2$ with different Θ and Φ angles. So check carefully that you are not submitting the same solution with a different orientation.

You can see it as my D14 that has faces oriented in the directions of the 6 faces and the 8 vertices of a cube where you would have replace two opposite faces by two pairs of faces (the kind C). By doing so, the faces corresponding to the vertices of the original cube are moved (the kind C) while the one corresponding to the faces of the cube that have not been doubled remain unchanged (kind A and D).

The separation I was speaking before is an important parameter. This is the angle at the vertex of the cone that has as a basis one face and at the vertex the center of the die. If r_2 is the distance from one face to the center and r_1 the radius of the sphere then the separation is $2 \cdot \arccos(r_2/r_1)$. Here the separation is $49,47^\circ$ approximately (for a distance between two parallel faces of 20.00mm, the diameter of the sphere is 22.02mm).

You have until September 30th to submit a design by answering this post. Tom will decide who the winner is if we have one. Note that by participating to this contest you accept that your design can be reused and modified by me or by Impact! Miniatures for our own usage, the compensation being the prize of the contest (\$100 in prepaid gift coupon).

Good luck!

Subject: Re: D16 Contest

Posted by [Magic](#) on Tue, 11 Sep 2012 06:06:07 GMT

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Hi there,

I have begun to receive some messages via PM.

Here are two tips to better "see" the structure of your polyhedron without having to draw it, in case you work with the coordinates of the centers of the circular faces on the (unit) sphere or if you

prefer the coordinates of the normal vectors of the faces.

Having your vector (x, y, z), to calculate Theta and Phi, you can use these formula:

- Theta = $\text{acos}(z)$

- Phi = $\text{atan2}(y,x)$

(do not forget to convert in degrees)

Another thing that can be useful, is to make the dot product between all the vectors taken two by two, because it gives the cosinus of the angle between two vectors. Thus it is easy to see if some directions are perpendicular or if you find some angles similar to my solution.

To calculate alpha the angle between vectors (x1,y1,z1) and (x2, y2, z2) use this formula:

- alpha = $\text{acos}(x1.x2+y1.y2+z1.z2)$

Let me know if you got some new D16!

Subject: Re: D16 Contest

Posted by [mctrivia](#) on Tue, 11 Sep 2012 15:06:34 GMT

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my best answer so far is 48.440809062632 deg of separation. I will keep looking for better though.

Subject: Re: D16 Contest

Posted by [mctrivia](#) on Wed, 12 Sep 2012 00:52:06 GMT

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49.6477401576 so I have finally beat you though barely. now to check all your other criteria.

Subject: Re: D16 Contest

Posted by [mctrivia](#) on Wed, 12 Sep 2012 04:04:06 GMT

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so what classifies as evenly distributed? 51.194013858768deg spacing is best I have found. This as a physical averaging that should result in fair but not the nice cosmetic even spacing that yours shows.

If this disqualifies me I have at least learned that different starting points can turn up better distributions. I will let my server farm run on this problem for a week and then draw up the best result what ever your ruling on the evenness of my design.

Subject: Re: D16 Contest
Posted by [Magic](#) on Wed, 12 Sep 2012 06:15:35 GMT
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More than 51° is very good, congrats. The best known for 16 points (without the constraint of parallel faces) is 52.2443957° (see this page).

I think the other important criterion is having an evenly distributed spherical area. I confess that, for me, this often means "as many symmetries as possible", but in a more rational approach it could also means "minimizing the largest round area".

I guess a mathematical definition would be "Once your equal circles are packing the sphere, the maximum radius of any extra circle you could insert between them should be as small as possible".

But at this point, trying to visualize the die is probably the best option.

Note that if you have "quasi-symmetries" (like an angle at 89.57°) you could enforce them (by forcing the angle to be 90° exactly).

Good luck!

Subject: Re: D16 Contest
Posted by [mctrivia](#) on Wed, 12 Sep 2012 21:45:45 GMT
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i seem to be maxing out at 51.3372888793 deg. pretty close to best known. Interesting but it sure takes a lot of computational power the way I am doing it. It takes me approximately 50000 iterations of my physics engine to compute one possible outcome(about 10 sec). I have my server trying 12 parallel attempts, running 24/7. In 1 week I will have done 36,288,000,000 iterations.

I have only found a 4deg improvement in total, and in the last 24 hours have not even gone up 0.2 deg.

Subject: Re: D16 Contest
Posted by [Magic](#) on Fri, 14 Sep 2012 18:47:57 GMT
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Wow! Your computation power is quite impressive...
I am curious to see how the "51.33Å,Å°" die looks like.
Let us know your progress!

Subject: Re: D16 Contest
Posted by [mctrivia](#) on Fri, 14 Sep 2012 22:15:37 GMT
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51.3613415955 is best I can get. Yes my server farm is pretty powerful. last i checked had over 1000 computer running in it. Will design the model and see what it looks like.

Subject: Re: D16 Contest
Posted by [mctrivia](#) on Sat, 15 Sep 2012 13:34:24 GMT
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51.36 deg separation. not as pretty as yours though. i think yours will be the most cosmetically appealing solution. THE dual looked so pretty guess the minor differences get exaggerated on the sphere.

File Attachments

1) [best.pdf](#), downloaded 98 times

Subject: Re: D16 Contest
Posted by [Magic](#) on Sat, 15 Sep 2012 16:02:32 GMT
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I must say I was hoping that some symmetries would appear...
Can you upload it on Shapeways so we can better see the model in the 3D view, please?
I am unsure but I think there is a strip of 8 faces touching each other around the equator, four faces on the top and four faces on the bottom.
In this case, the 8 faces are the ones that are determining the "separation". If they were all on the equator forming a regular octagon, it would not be very efficient. We have to move some of them

up and some of them down.

The best configuration I see is the following one ("+" means up and "-" means down for each vertex of the octagon):

..+..-..

-.....+

-.....+

..+..-..

Perhaps we should try to initialize the system with such a configuration and see if we end up with something interesting...

Subject: Re: D16 Contest

Posted by [mctrivia](#) on Sat, 15 Sep 2012 17:36:15 GMT

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http://www.shapeways.com/model/702067/d16_temp.html?key=dfaf d0c7bcfb5b6819b4e26157e3e5b6

that is a 3d pdf. click on the shape then you can rotate it anyway you like.

will have to make some modifications to let me set the start point.

Subject: Re: D16 Contest

Posted by [impactminiatures](#) on Sat, 15 Sep 2012 17:38:26 GMT

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McTrivia,

Wow I didn't know you could make turnable image PDFs. I found that amazing good to really look over what you had made. I'm wishing Shapeways had this option now for each product ... would save me a lot of time trying to figure out the designs I'm looking over!

Magic ... after spinning McTrivia's version around ... the pattern in it looks to me like it is 2-6-6-2. Now the question is if this is "better" than the 1-4-6-4-1 design (I should note for comparison that Magic's dice could be turned in a way that the design is 2-4-4-4-2 (to my non-math mind trying to compare and contrast the dice designs)).

McTrivia ... I've followed a lot of your designs on Shapeways. Your talent is impressive as well.

So let me get your thoughts. If you were personally picking a mass production version of the D16 ... would you favor the 2-6-6-2, 1-4-6-4-1 (which both have numbers on the faces) ... or Magic's version with the numbers on the edges:

My concern is this ... from our last KickStarter ... we had a lot of folks call out that even sided dice should have numbers on the faces ... they were fine and understood this not being the case for more true odd sided dice.

I will admit in getting acceptance of the dice by the masses ... being able to say that 36.3 billion computations were done with the physics computer to get the optimal shape. Goes a long way to arguing the fairness equation.

I will say ... I personally will be happy that if the two of you come to a consensus on what is the right die to make for the masses ... that I am very willing to listen to that advice.

The end goal of our next project is to make a DCC set that has a uniform color scheme and is spherical in nature for the whole set. The D16 is one of those we need to get nailed down for the project to work.

Subject: Re: D16 Contest
Posted by [mctrivia](#) on Sat, 15 Sep 2012 18:02:43 GMT
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What is best is a matter of what is the use. Both i suspect are fair designs. Magic's strict mathematical approach comes up with some beautiful intelligently designed dice this being in my opinion one of the best non-platonic designs. Myself I am using chaos and particle physics to come up with a design that should be fair and have the widest possible separation of axes(sets of faces). The advantage of my approach is in 10 sec I can come up with the dual of any even number sided die and with multiple iterations fine if not the best very close to it separation possible. Magic's method may not be ideal separation but will almost always come up with a more cosmetically appealing approach.

Personally if I was in the need of a D16 for just a general use then I would pick Magic's and I suspect most would. I can see mine be a cool and different addition to a board game along with several other chaotically designed dice.

Subject: Re: D16 Contest

Posted by [impactminiatures](#) on Sat, 15 Sep 2012 18:17:08 GMT

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So for someone wanting a quick image comparison ... here is McTrivia's die from views similar to the ones Magic has above.

I will note ... I can see how the overall rounded space is less with this version than Magic's. I can also now see that it is done at the cost of looking equally spaced from the 3 different views that Magic has of his version.

Tough one to call. The uniform look when cut in half on 3 axis of Magic's version has an appeal to non-math folks like me ... but I also like knowing the round space is minimal from McTrivia's.

I will be interested to hear from you guys on your future thoughts on this.

Thank you both for looking at this project.

Tom @ Impact!

Subject: Re: D16 Contest

Posted by [impactminiatures](#) on Sat, 15 Sep 2012 18:26:40 GMT

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mctrivia wrote on Sat, 15 September 2012 18:02 Myself I am using chaos and particle physics to come up with a design that should be fair and have the widest possible separation of axes(sets of faces).

Personally if I was in the need of a D16 for just a general use then I would pick Magic's and I suspect most would. I can see mine be a cool and different addition to a board game along with several other chaotically designed dice. You have a good point with this McTrivia. My problem is that I love chaos so your design has an appeal to me.

What we could definitely do is put it to a vote when the KickStarter hits the point of funding the D16 and give the backers the choice between the two versions. Explain that one is equal dimensioned on 3 axes and that the other is designed with chaos physics to have the minimal amount of rounded space (but I think not equal on any axes? (that I could see)). I'm happy to produce the one that the backers vote for.

If you make the change that Magic requested ... I would really be interested in seeing what comes of that modification.

Tom @ Impact!

Subject: Re: D16 Contest
Posted by [Magic](#) on Sat, 15 Sep 2012 19:42:08 GMT
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Hi Tom and McTrivia (and all the others),

I am not at home currently, but I managed to access an old version of the software I wrote to optimize dice.

Taking Tom's notation, I've done a 2-6-6-2 configuration (or more precisely a 2-2-4-4-2-2, because you can add some more degrees of freedom by letting 2 of the 6 faces of the second layer being a little bit shift relatively to the others) in the following way:

- A1, A2 on top
- B2, B5 on an axis perpendicular to A1, A2
- B1, B3, B4, B6 touching the equator, so that B3 is symmetric to B4 relatively to the A1-A2 axis and B1 is symmetric to B6

The best I get is 44.62° of separation. The more recent version of my software will probably give a better result, but still this is a very poor value for the separation.

And unfortunately I cannot visualize it currently.

More news from me tomorrow night, I guess...

PS: I did not notice this was a 3D PDF
It is very handy indeed.

Subject: Re: D16 Contest
Posted by [mctrivia](#) on Sun, 16 Sep 2012 02:13:45 GMT
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Bad news. found an error in my physics engine which invalidates all my previous results. have started rerunning the engine but down to only 48.405834944358

Subject: Re: D16 Contest
Posted by [mctrivia](#) on Sun, 16 Sep 2012 15:04:47 GMT
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magic are you sure you have 49deg separation? I did not recreate your point cloud to check but instead used your sphere dimensions.

As I calculate it a sphere 22.02mm with faces 20mm apart results in a face 4.6065mm in radius. this gives an angle from center to edge of the circle of 12.97deg. so your minimum separation angle is only 25.94deg

if my math is correct that makes my 48.405834944358(current best after fixing math error), 1.86 times better spaced.

Please check these discrepancies. I did check 0.01mm error factor in your numbers only increases to 26deg

Subject: Re: D16 Contest
Posted by [Magic](#) on Sun, 16 Sep 2012 15:42:01 GMT
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4.6065mm for the radius of the face is correct, but I don't undersand how this leads to 12.97°,°.

The calculation I did is the following one:

- distance from center to face = 10.00mm (20.00 / 2)
 - radius of the sphere = 11.01mm (22.02 / 2)
 - separation = $2 \cdot \arccos(10.00/11.01) = 49.627^\circ$
- You got the same result wih $2 \cdot \arcsin(4.6065/11.01)$.

Note that for the sphere, I always use a radius slightly smaller than the exact value to be sure that the faces are properly separated even if the sphere is only an approximation of sphere. So 49.467° is the actual value used for the model, but from my calculations the theoretical value should be rather 49.627° .

Subject: Re: D16 Contest
Posted by [mctrivia](#) on Sun, 16 Sep 2012 15:53:57 GMT
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a triangle with 20mm and 4.61mm sides has 13deg angle.

Subject: Re: D16 Contest
Posted by [Magic](#) on Sun, 16 Sep 2012 16:17:58 GMT
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Yep. But here, the distance from face to center is 10mm only (20mm is from face to face).

Subject: Re: D16 Contest
Posted by [mctrivia](#) on Sun, 16 Sep 2012 16:43:15 GMT
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there we go that works out now.

So far it looks like your arrangement may very well be the ideal.

Subject: Re: D16 Contest
Posted by [mctrivia](#) on Sun, 16 Sep 2012 23:46:47 GMT
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after ten billion computations I am 99.99999% sure yours is the best possible solution Magic. Will have to get one of these dies.

Subject: Re: D16 Contest
Posted by [impactminiatures](#) on Sun, 16 Sep 2012 23:50:01 GMT
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mctrivia wrote on Sun, 16 September 2012 23:46after ten billion computations I am 99.99999% sure yours is the best possible solution Magic. Will have to get one of these dies.

mctrivia ... you my good man are awesome. That is a great line to use for the KickStarter when we do it.

Magic ... that is enough reassurance for me. Go ahead and put numbers on your numbers on the faces D16 and we'll run with that design!

Tom @ Impact!

Subject: Re: D16 Contest
Posted by [Magic](#) on Mon, 17 Sep 2012 06:13:32 GMT
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McTrivia, I do appreciate the compliment, in particular coming from a great dice designer like you. Thanks.

Tom, I'll work on the numbering of this D16 as soon as possible, but I keep the contest open until September 30th, in case someone else would like to participate.

To all, I should not be available to answer any post from 20th to 30th of September (unless I find an Internet access where I will be), but do not hesitate to post any solution you think could compete: I will have a look at it carefully at my return.

Thank you all!

Subject: Re: D16 Contest
Posted by [aleakybos](#) on Sat, 22 Sep 2012 16:19:10 GMT
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Magic & McTrivia, great work! It is always fascinating if a design "by construction" also turns out as optimum. Genius at work

There is little to add from my side, as an old Dell PC can not compete with McTrivia's impressive server farm. Nevertheless, I can confirm that Magic's solution is indeed at least a local optimum, and I have not found anything better. However, if Magic's solution is indeed optimum, then I claim that there are infinitely many optimum solutions!

I tried to visualize the constellation of the points and the corresponding polyhedron with the applet on my home page:http://www.aleakybos.ch/sph_codes.htm : choose PackAnti from the drop down menu (packing optimization criterion, antipodal = parallel faces) and 16 points.

Point 0 is the North pole (A1 is Magic's description), red lines correspond to the minimum distance

between any two points. It can be seen that the points on the second layer (4,5,6,7 or B1-B4) all have minimum distance to the North pole and also to one of their two neighbors on the same layer (4-5 and 6-7). Layer 3, the equatorial plane, is interesting. Of the 6 points, there are two pairs with minimum distance to each other (e.g. 3-10) and to two points in the upper layer (3-5 and 10-7). However, there are also two points (1 and 9 or D1 and D2 in Magic's picture) that are so called rattlers: the circles on these points do not touch any other circle! This means that point 1 (and its antipode 9) can be moved inside a small area (about 1.4% of the spheres' radius) as long as the distance to its neighbours is not below the minimum distance, hence there are indeed infinitely many designs, Although I have not found a design that beats Magic's, I found many that are equally good

However, it makes sense to allocate points 1 (and 9) such that the design is symmetric, so kudos to Magic once more!

No worry, Tom, no need to produce infinitely many designs, one will do

Given those symmetries, it is actually possible to express the coordinates analytically. Assuming that Theta is the elevation of the first plane (same notation as Magic) and Beta is the smaller azimuth angle between two points on the first plane (4-5 or B1-B2), it can be shown that $\cos(\text{Beta}) = (\cos(\text{Theta}) - \cos^2(\text{Theta})) / (1 - \cos^2(\text{Theta}))$

and

$$\cos(\text{Theta}) = \sin(\text{Theta}) * \sin((\text{Theta} + \text{Beta}) / 2)$$

Solving these two equations numerically yields

$$\text{Theta} = 49.62703013^\circ$$

$$\text{Beta} = 66.85149262^\circ$$

This relates to Magic's design as follows:

Second and 4th layer: $\text{Phi} = \pm \text{Beta} / 2$ and $\pm (180^\circ - \text{Beta} / 2)$

Third layer, points C: $\text{Phi} = \pm (90^\circ - \text{Theta} / 2)$ and $\pm (90^\circ + \text{Theta} / 2)$

So much for now.

Alea Kybos

File Attachments

1) [D16 symmetries.jpg](#), downloaded 480 times

Subject: Re: D16 Contest

Posted by [impactminiatures](#) on Sat, 22 Sep 2012 17:16:54 GMT

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aleakybos,

Great post ... very interesting reading even for a non-mathematician.

I took a stab at numbering the D16 and man even spending 3 hours on it ... I have a feeling Magic will show me a better way. So I have a lot of respect for the dice designers of Shapeways.

I think we the help we are getting the new dice that Impact! will be producing will be an awesome addition to the gaming community!

Subject: Re: D16 Contest

Posted by [Magic](#) on Fri, 28 Sep 2012 15:44:17 GMT

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McTrivia and AleaKybos, thank you for your contributions. There are still 2 days left for the contest, so do not hesitate to post any idea.

AleaKybos, your calculations are really interesting, thanks again. I know you are a fan of Truncated Sphere Dice, it is always a pleasure to dicuss with you about this topic . I've seen your new PackAnti option. It is really awesome. But for 22, there is no clear symmetric solution... I was wondering if you could add a "CoveringAnti" (or add an "anti" checkbox so that we could use any optimisation algorithm with the parallel faces constraint). I am curious to see the result for other even faces dice...

Cheers,

Magic

Subject: Re: D16 Contest

Posted by [aleakybos](#) on Thu, 04 Oct 2012 17:50:14 GMT

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Magic, thank you for your feedback. Actually, the applet on my home does not compute dice, it just displays known configurations. So, if you would like to add one of the dice to the applet, just send me the coordinates and I will upload the new design to visualize it.

McTrivia: Some of your truncated spherical dice in my collection seem to have parallel faces, the

D22, D28, D30, D40, D60, and D144 (although there I am not completely sure, the faces are so small). I assume that you calculated them the same way as the D16 in this contest, with your huge server farm. Would you mind sharing your D22 design with me, the applet might reveal some symmetries.

Of the optimum configurations in the sense of maximum separation (or "packing"), only 2 have parallel faces: the D6 (the good old cube), D12 (pentagonal dodecahedron). The optimum D8 is not the octahedron, but the 4-antiprism, which has no parallel faces.

So, there are more spherical dice with parallel faces to be designed, e.g. the D8 (is this the octahedron?), D14, D18 and D20 (is this the dodecahedron?)

Subject: Re: D16 Contest
Posted by [mctrivia](#) on Fri, 05 Oct 2012 11:50:08 GMT
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all my newer spherical dice with even faces of parallel faces. I computed them in the same fashion though I did not take the time to guarantee optimality just close. Unfortunately the computer I designed those dice on died a while back and took the point clouds with it. I have a much better backup system in place now to guarantee that never happens again but it is too late to recover those dice.

Subject: Re: D16 Contest
Posted by [Magic](#) on Sat, 06 Oct 2012 09:12:48 GMT
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AleaKybos, I will send you the coordinate of the D16 by PM. But I think there are the same as your configuration.
I'd like also to see if we get for the D14 and D18 is the same configuration as the ones I already designed.
For the D20, I must say that there is a lot of round areas for the icosahedron, because there are always 5 faces meeting at each vertex. So perhaps we can do better. And the good thing is that, by trying to design the D16, I found a lot of ways to make a D20 (because very often I obtained 16 faces and 4 by round areas that could contain themselves a face after some deformation)...

McTrivia, if you re-write your program, it will be interesting to see if we can find symmetries in a dice that begins with random positions.

Thanks to both of you.

To all, the contest is finished! I hope you will soon see the D16 numbered in my Shapeways shop, and later mass-produced by Tom from Impact! Minatures !
I'll keep you updated!

Subject: Re: D16 Contest
Posted by [tos Crawford](#) on Tue, 09 Oct 2012 00:33:16 GMT
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While I appreciate the creativity required to build the perfect die, with the D16 I'm not sure you know what you have. This is way bigger than a a D16. You have also invented the 2D4 die.

After you've got the die just the way you want it, build one with numbers
2,3,3,4,4,4,5,5,5,5,6,6,6,7,7,8 for me please.

Subject: Re: D16 Contest
Posted by [impactminiatures](#) on Tue, 09 Oct 2012 00:50:15 GMT
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I'm sure Magic could design one of those for Shapeways ... and we can put it in the long term cue to try and build.

Tom

Subject: Re: D16 Contest
Posted by [Magic](#) on Tue, 09 Oct 2012 21:25:27 GMT
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So many things to design, so little time... (did I already say that?)
Tos Crawford, you are right: each time we design a dice with a number of faces that is a square number, we can simulate two dice with this one. I will do this numbering for you if you wish (that's the beauty of 3D printing, you can personalized objects as you wish). But, if you cannot wait, McTrivia already did this kind of numbering for its own D16 called the Dual D4

Subject: Re: D16 Contest

Posted by [tos Crawford](#) on Wed, 10 Oct 2012 02:23:33 GMT

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Thanks for pointing out the dual d4. Its cool that all this is possible. In the case of the dual d4, I'm not really a fan of how the numbers are placed. For the time being I think I'd rather wait to see how yours comes out. I'm definitely buying something, but I have the luxury of time before making my decision.

Subject: Re: D16 Contest

Posted by [Magic](#) on Sat, 13 Oct 2012 17:48:03 GMT

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As you wish. I will make this double-D4 numbering as soon as possible (with opposite summing to 10 and numbers as well balanced as possible).

If you like the idea of combining two dice in one, you will perhaps also like this two models:

- the Double Coin D4
- the Double D6

Unlike the D16, they have moving parts though.

I keep you updated.

Subject: Re: D16 Contest

Posted by [aleakybos](#) on Tue, 16 Oct 2012 05:07:54 GMT

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Magic, wow, the Double Coin D4 and the Double D6 are cool designs, keep going!

I am still working on the D14 and the D18. For the D14 I haven't found anything better than your "cuboctahedron" yet ($\phi = 54.74^\circ = \arccos(1/\sqrt{3})$).

For the D18, however, I found a design that is better than your Rhombi-cuboctahedron. It is very irregular, and I am sure that by a combination of manual and computational optimization it can still be improved. I will keep you posted.

Subject: Re: D16 Contest

Posted by [Magic](#) on Tue, 16 Oct 2012 20:15:18 GMT

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Thanks AleaKybos. Can't wait to see this D18.

Toscrawford, the D16 numbered as two D4 is now available:

Enjoy!

Subject: Re: D16 Contest
Posted by [tos Crawford](#) on Tue, 16 Oct 2012 22:35:46 GMT
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That's awesome. Thanks!

Subject: Re: D16 Contest
Posted by [aleakybos](#) on Fri, 19 Oct 2012 20:07:34 GMT
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Magic, that D16 2D4 looks great!

As promised, this is my D18, designed with maximum separation ("packing") and parallel faces. The angle is 47.9821 deg, compared to 45deg of Magic's D18 based on the squares of Rhombicuboctahedron.

You may play with the applet here to see the symmetries (PackAnti, 18)
http://www.aleakybos.ch/sph_codes.htm

There is a chance that someone may find an even better angle, as the design was done by a numerical search with some analytical optimization once the symmetries were guessed. If you find a better design, please let me know.

Cheers, Alea Kybos

PS. McTrivia, I put quite some effort into optimizing my simulation, as I do not have access to a huge server farm like you. A brute force approach for a D2N would use 6N cartesian or 4N spherical coordinates, whereas I brought it down to 2N-3 spherical coordinates and each

evaluation of the objective function uses $2N-3$ sine and cosine operations and $O(N^2)$ multiplications. Still a lot...

File Attachments

1) [D18AK.jpg](#), downloaded 208 times

Subject: Re: D16 Contest

Posted by [Magic](#) on Sat, 20 Oct 2012 07:01:15 GMT

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Hi AleaKybos,

Thank you for this new polyhedron!

I played with the applet and I have a concern: all seems symmetric relatively to two perpendicular planes except two faces... I am wondering why these skewed faces would give a better result than faces that would be parallel to the axes...

In this drawing, if the top and bottom faces were horizontal, I think it would help.

What do you think?

File Attachments

1) [D18Alea.jpg](#), downloaded 182 times

Subject: Re: D16 Contest

Posted by [Magic](#) on Sat, 20 Oct 2012 07:25:35 GMT

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Oh, actually, it makes sense.

Look at the dice without these two skewed faces and with them...

They perfectly fit inside the empty space!

Very interesting: it proves that the intuition that symmetry is always better is wrong in some cases.

[EDIT] Here is the link to the model: <http://www.shapeways.com/model/745108>

The animation where the die rotates around the Z axis helps to understand how the faces are located and why the skewed position for the last two ones is the best...

File Attachments

1) [D18AleaKybosBoth.jpg](#), downloaded 199 times

Subject: Re: D16 Contest

Posted by [aleakybos](#) on Sun, 21 Oct 2012 09:11:09 GMT

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Magic, thank you very much for the design! Yes, the "poles" are shifted compared to the other layers. I therefore rotated the die by 90 deg by setting the pole half way between points 0 and 1 and constructed the die from there.

Here comes another confirmation of your statement "the intuition that symmetry is always better is wrong in some cases". After having designed the D18, I ran my tools again over your D16, and, oops!, after some minutes a "better" solution popped up. I have kept my PC busy over the last 36 hours and I still keep getting better solutions, although progress is 4 digits after the comma... Current status is 49.63899330 deg compared to 49.62703013 deg.

My explanation is that in your design there are 2 rattlers (points that do not have minimum distance to any neighbor), as explained in my earlier post. Now there seems to be an even better solution if those neighbors are shifted closer towards the rattler and to distribute the "gained angle" over the rest of the configuration, thereby increasing the minimum distance (separation). The final version may thus no longer have the same symmetries. I uploaded the current best solution in my applet 16 Points, PackAnti), there you see the configuration (coords and angle are in the report window).

Anyway, the difference to the contest winner is tiny, only about 0.02%, i.e., way below production tolerances. So this analysis is purley "alealogic" ... I will keep you posted.

Subject: Re: D16 Contest

Posted by [Magic](#) on Mon, 22 Oct 2012 21:59:28 GMT

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This improvement is really interesting indeed (from the theoretical or "alealogic" point of view). Unfortunately, I am afraid that we will not notice the difference with the naked eye. That's a pity.

On my side, I have worked on another D16 with tetrahedral symmetry (not with numbers on faces, though), and I had to deal with "rattlers" too. You will find the story here.

I did not checked yet, but I am sure we can find at least one of them with your program...

Subject: Re: D16 Contest
Posted by [Magic](#) on Sun, 28 Oct 2012 11:25:53 GMT
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The D18 discovered by Alea Kybos is now resized, hollowed, numbered and available for sale...

Subject: Re: D16 Contest
Posted by [Magic](#) on Sat, 08 Dec 2012 19:54:13 GMT
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Alea Kybos was kind enough to send me these pictures of the D18 he designed and ordered in metal.

Here are two pictures:

The result in metal is fantastic.

And on these pictures the asymmetry of the "18" face is clearly visible.

Thank you Alea Kybos!

Subject: Re: D16 Contest
Posted by [Magic](#) on Tue, 14 Jan 2014 10:21:59 GMT
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The discussed D16, as well as some other Sphere Dice, is about to be mass produced: check this thread!